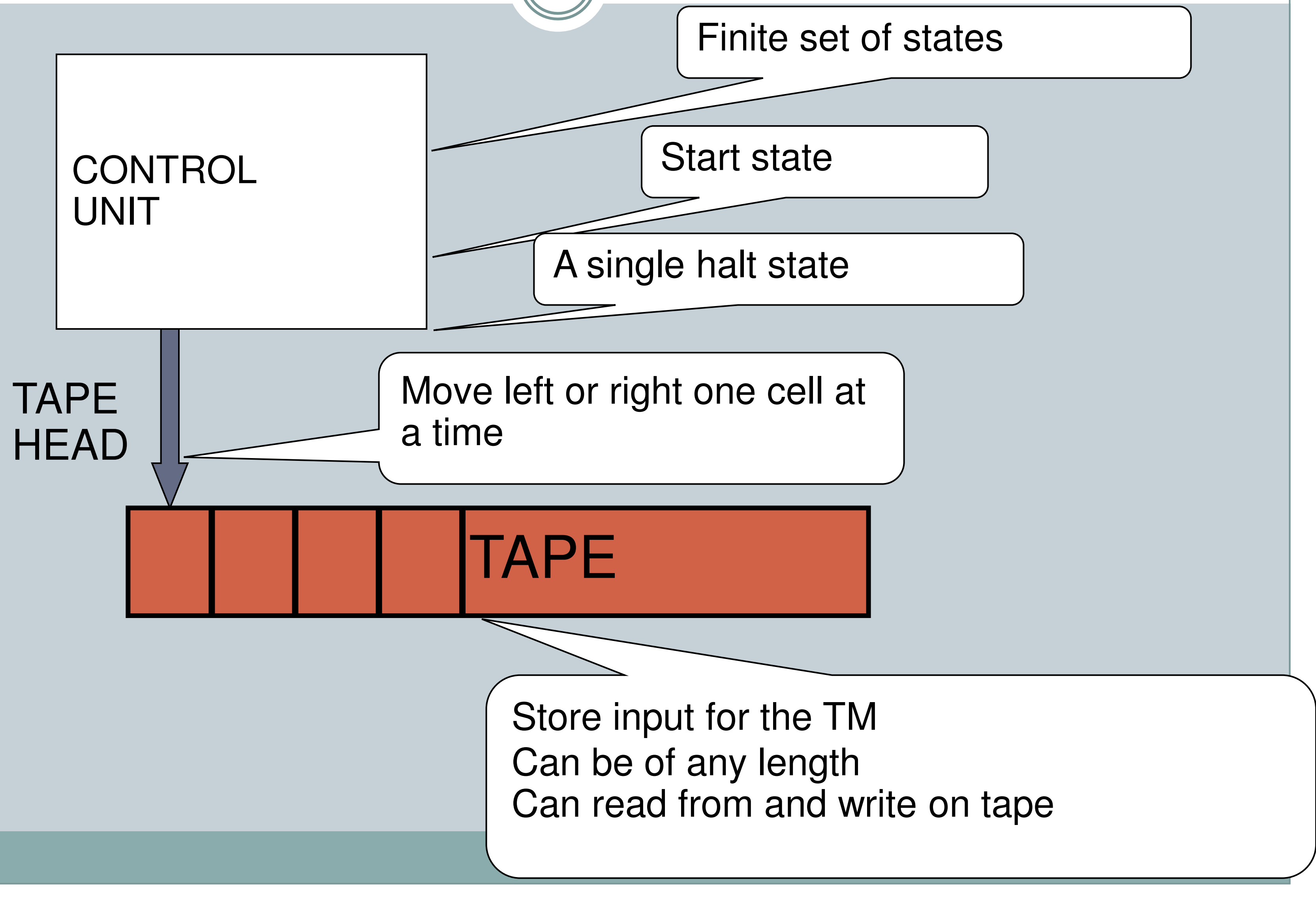
Turing Machines (TM)

MODEL OF COMPUTATION

Outlines

- Structure of Turing machines
- Deterministic Turing machines (DTM)
  - Accepting a language
  - Computing a function
- Composite Turing machines
- Multitape Turing machines
- Nondeterministic Turing machines (NTM)
- Universal Turing machines (UTM)

Structure of TM



What does a TM do?

- Determine if an input x is in a language.
  - That is, answer if the answer of a problem P for the instance x is "yes".
- Compute a function
  - Given an input x, what is f(x)?

How does a TM work?

- At the beginning,
  - A TM is in the *start state* (*initial state*)
  - its tape head points at the first cell
  - The tape contains Δ, following by input string, and the rest of the tape contains Δ.

How does a TM work?

- For each move, a TM
  - reads the symbol under its tape head
  - According to the *transition function* on the symbol read from the tape and its current state, the TM:
    - write a symbol on the tape
    - move its tape head to the left or right one cell or not
    - changes its state to the *next state*

When does a TM stop working?

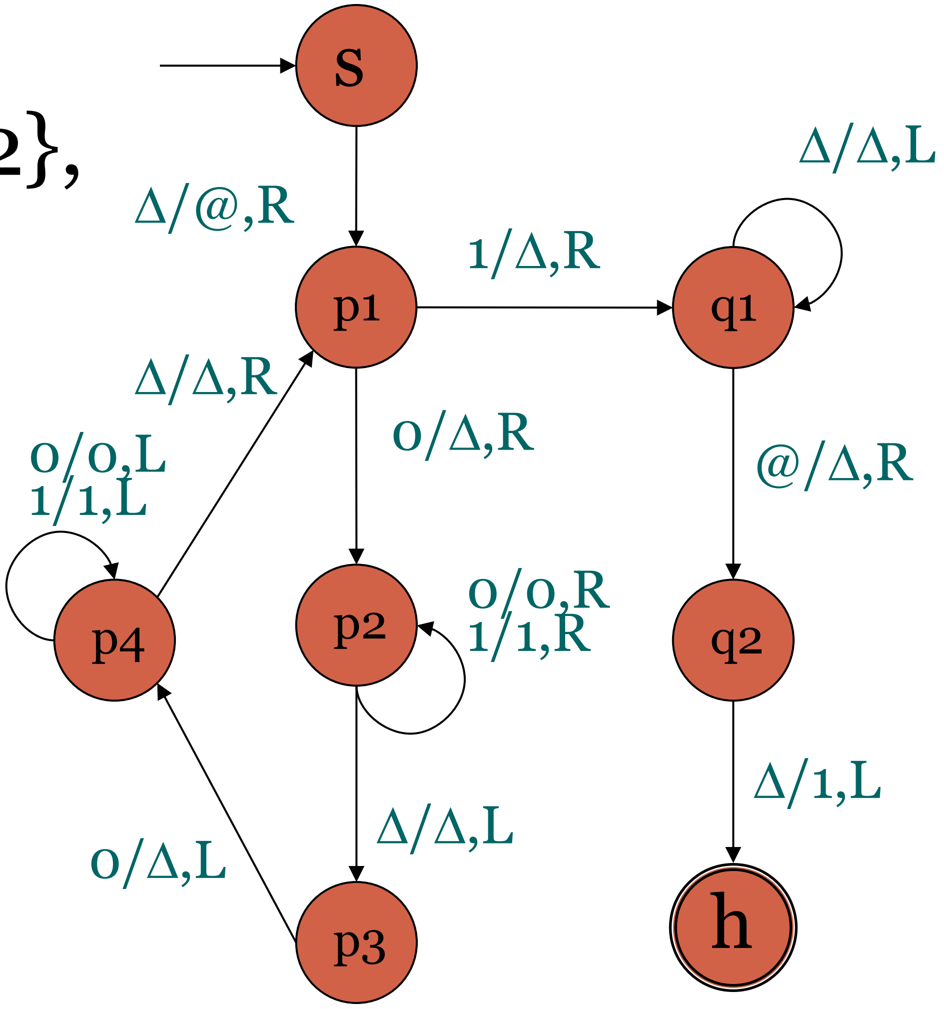
- A TM stops working,
  - when it gets into the special state called halt state. (halts)
    - The output of the TM is on the tape.
  - when the tape head is on the leftmost cell and is moved to the left. (hangs)
  - when there is no *next state*. (hangs)

How to define deterministic TM (DTM)

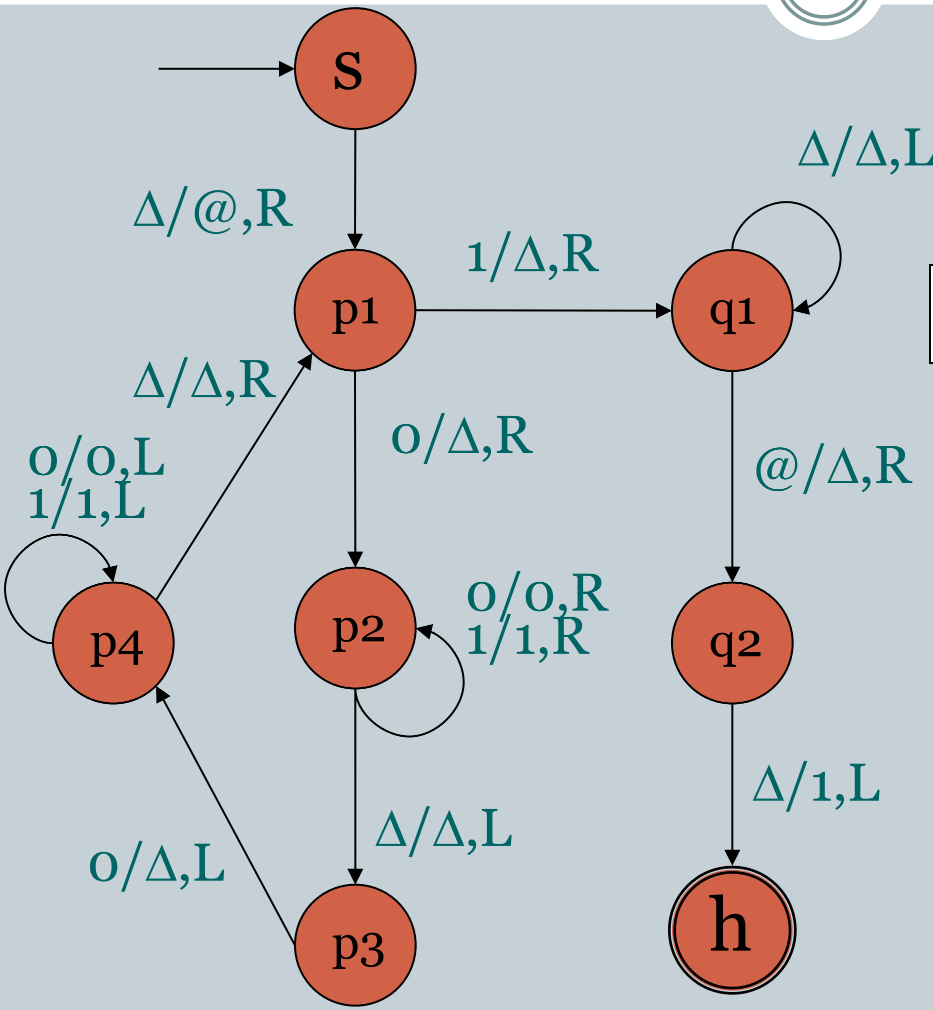
- a quintuple (Q, Σ, Γ, δ, s), where
  - the set of states Q is finite, not containing halt state h,
  - the input alphabet Σ is a finite set of symbols not including the blank symbol Δ,
  - the tape alphabet Γ is a finite set of symbols containing Σ, but not including the blank symbol Δ,
  - the start state s is in Q, and
  - the transition function δ is a partial function from $Q \times (\Gamma \cup \{\Delta\}) \rightarrow Q \cup \{h\} \times (\Gamma \cup \{\Delta\}) \times \{L, R, S\}$.

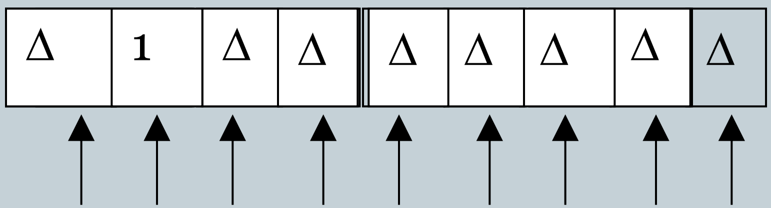
Example of a DTM

- M= ({s,p1,p2,p3,p4,q1,q2}, {0,1},{0,1,@}, δ, s}



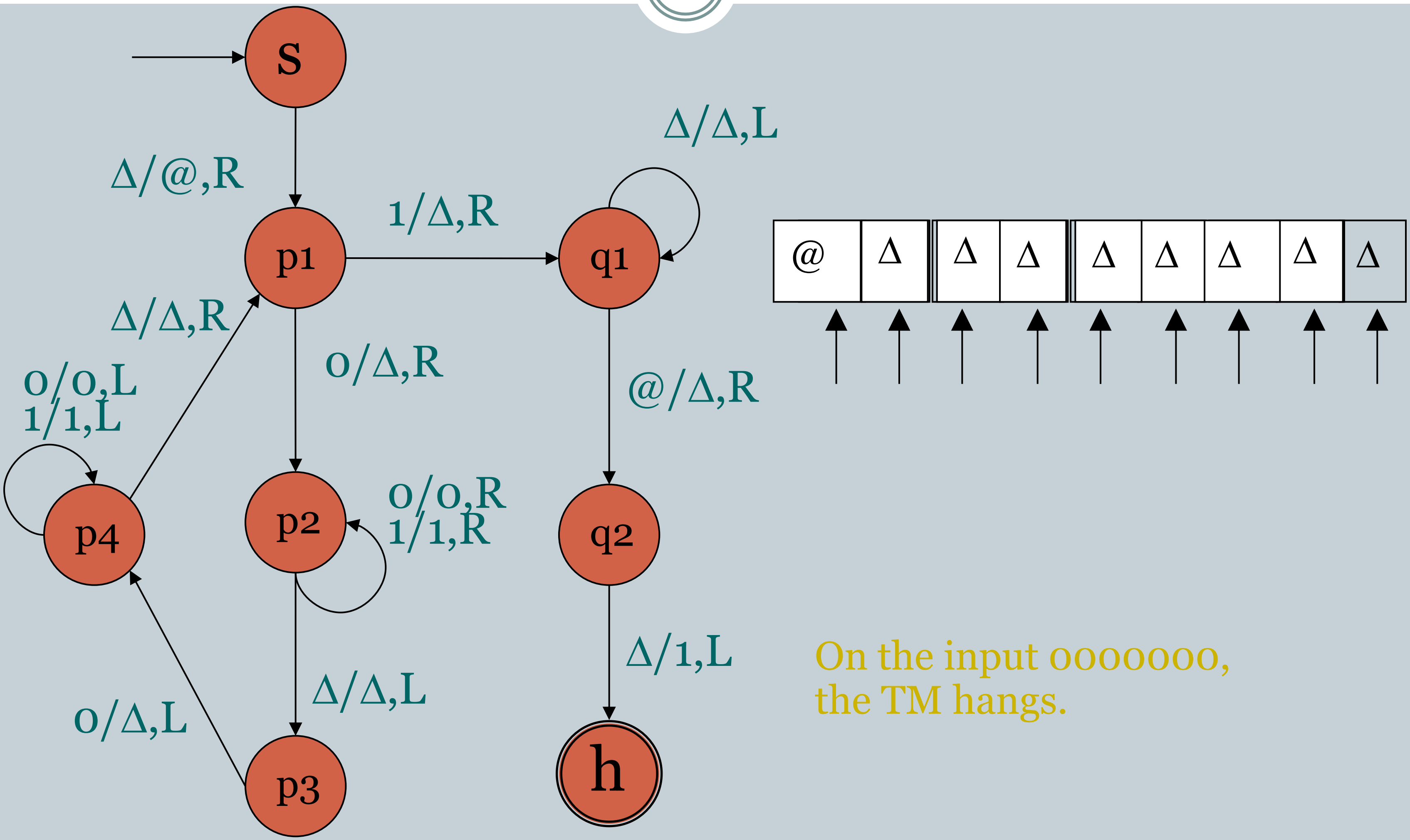
How a DTM works





On the input 0001000,
the TM halts.

How a DTM works



On the input 0000000,
the TM hangs.

Configuration

Definition

- Let $T = (Q, \Sigma, \Gamma, \delta, s)$ be a DTM.

  A configuration of T is an element of

$$Q \cup \{h\} \times (\Gamma \cup \{\Delta\})^* \times (\Gamma \cup \{\Delta\}) \times (\Gamma \cup \{\Delta\})^*$$

- Can be written as

Current state

string to the right of tape head

symbol under tape head

string to the left of tape head

Yield the next configuration

Definition

- Let $T = (Q, \Sigma, \Gamma, \delta, s)$ be a DTM, and $(q_1, \alpha_1\underline{a_1}\beta_1)$ and $(q_2, \alpha_2\underline{a_2}\beta_2)$ be two configurations of T.

  We say $(q_1, \alpha_1\underline{a_1}\beta_1)$ yields $(q_2, \alpha_2\underline{a_2}\beta_2)$ in one step, denoted by $(q_1, \alpha_1\underline{a_1}\beta_1) \vdash_T (q_2, \alpha_2\underline{a_2}\beta_2)$, if

  - $\delta(q_1, a_1) = (q_2, a_2, \mathrm{s})$, $\alpha_1=\alpha_2$ and $\beta_1=\beta_2$,
  - $\delta(q_1, a_1) = (q_2, b, \mathrm{R})$, $\alpha_2=\alpha_1 b$ and $\beta_1=a_2\beta_2$,
  - $\delta(q_1, a_1) = (q_2, b, \mathrm{L})$, $\alpha_1=\alpha_2 a_2$ and $\beta_2=b\beta_1$.

Yield in zero step or more

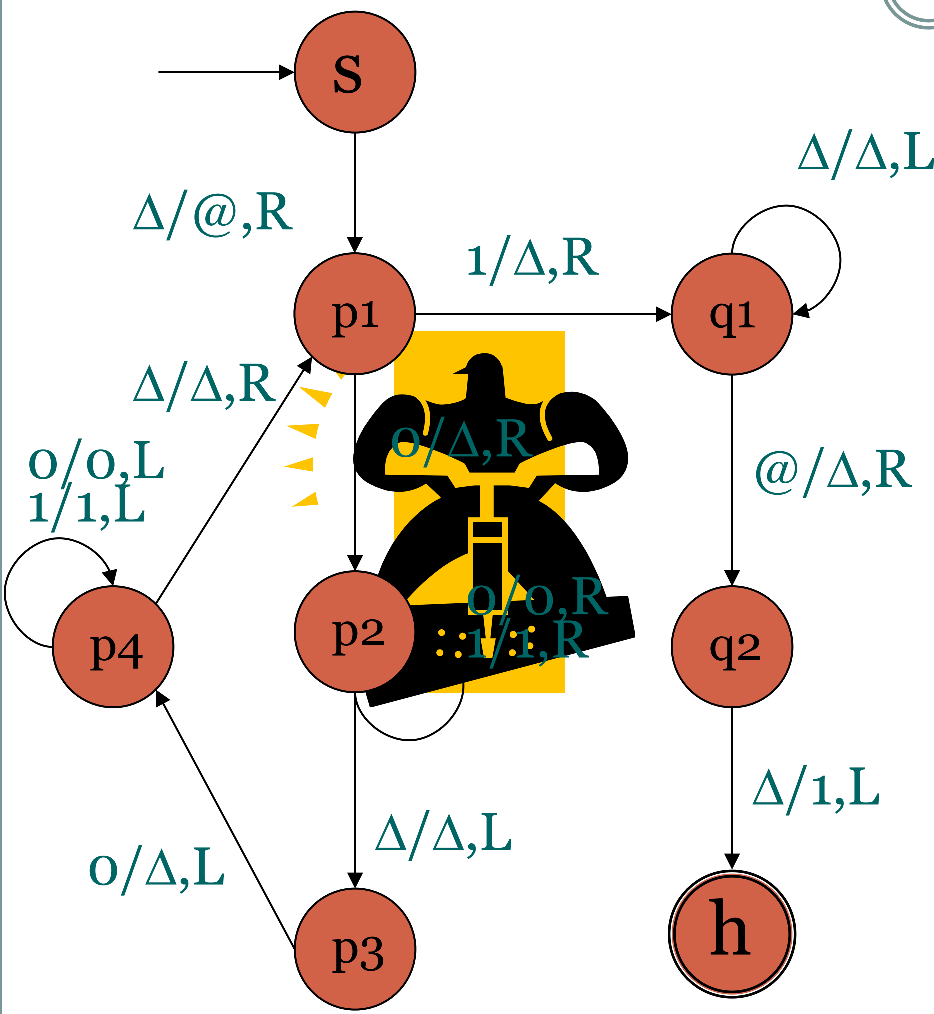
Definition

- Let $T=(Q, \Sigma, \Gamma, \delta, s)$ be a DTM, and $(q_1, \alpha_1\underline{a_1}\beta_1)$ and $(q_2, \alpha_2\underline{a_2}\beta_2)$ be two configurations of T.

  We say $(q_1, \alpha_1\underline{a_1}\beta_1)$ yields $(q_2, \alpha_1\underline{a_2}\beta_2)$ in zero step or more, denoted by $(q_1, \alpha_1\underline{a_1}\beta_1) \vdash^*_T (q_2, \alpha_1\underline{a_2}\beta_2)$, if

  - $q_1=q_2$, $\alpha_1 =\alpha_2$, $\underline{a_1}=\underline{a_2}$, and $\beta_1= \beta_2$, or
  - $(q_1,\alpha_1\underline{a_1}\beta_1) \vdash_T (q, \alpha\underline{a}\beta)$ and $(q, \alpha\underline{a}\beta) \vdash^*_T (q_2,\alpha_1\underline{a_2}\beta_2)$ for some q in Q, α and β in Γ^*, and a in Γ.

Yield in zero step or more: Example



(s,Δ0001000)
(p1,@0001000)
(p2,@Δ001000)
(p2,@Δ001000Δ)
(p3,@Δ001000)
(p4,@Δ00100Δ)
(p4,@Δ00100Δ)
(p1,@Δ00100Δ)
(p2,@ΔΔ0100Δ)
(p2,@ΔΔ0100Δ)
(p3,@ΔΔ0100)

(p4,@ΔΔ010)
(p4,@ΔΔ010)
(p1,@ΔΔ010)
(p2,@ΔΔΔ10)
(p2,@ΔΔΔ10)
(p2,@ΔΔΔ10Δ)
(p3,@ΔΔΔ10)
(p4,@ΔΔΔ1)
(p4,@ΔΔΔ1)
(p1,@ΔΔΔ1)
(q1,@ΔΔΔΔΔ)
(q1,@)
(q2,ΔΔ)
(h ,Δ1)